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OPTIMUM HVAC-TRANSMISSION EXPANSION PLANNING - A NEW FORMULATION

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Abstract - A new flexible mixed-integer formulation is presented in this paper for solving the single-stage ac-transmission expansion planning problem. This new formulation handles exactly the discrete nature of equipment additions without resorting to linearized incremental approaches, but still uses integer linear programming. The interconnection of a disconnected system is handled directly without resorting to an initial connected system. The optimal configuration at the end of the planning step under consideration minimizes an annually amortized cost function including the investment cost for equipment additions and the operating cost in terms of real-power transmission losses. Using a new efficient linear load-flow model, this new formulation is characterized by a complete decoupling between real and reactive equations. Examples have been incorporated to illustrate the potentials of this new optimum planning formulation.

INTRODUCTION

Planning the expansion of high-voltage power systems consists of choosing, over a period of several years, an optimal expansion policy. This policy specifies where, when and what kind of equipments, such as generation and transmission facilities, should be added. The resulting implementation strategy is optimal in the sense that investment and operating costs are minimized.

New equipments are implemented in discrete units, therefore only discrete optimization techniques must be used for solving the transmission expansion problem. The difficulties of this problem come from the tremendous number of possible alternatives, and the large number of constraints.

A new flexible mixed-integer formulation is presented in this paper for solving the single-stage ac-transmission expansion planning problem. Unlike earlier formulations [1-7] using sensitivity analysis together with integer programming, this formulation does not require the existence of sensitivity coefficients. In addition this new method offers the following advantages:

1. The discrete nature of equipment additions is handled exactly without resorting to linearized incremental approaches.
2. The complete set of admissible network reinforcements (including phase-shifters) is considered.
3. The interconnection of disconnected or disjoint systems can be handled directly without resorting to an initial connected system.
4. Linear and/or discrete cost-capacity curves can be used.
5. A new efficient linear load-flow model is incorporated in the method.

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Mixed-integer linear programming techniques are used for solving this single-stage transmission expansion planning problem. The new generation and load levels are given. It is also assumed that the right-of-ways have been selected. The optimal configuration at the end of the planning step under consideration minimizes a cost function which includes the total annually amortized investment cost for equipment additions and the total annual operating cost in terms of the cost of real-power transmission losses. The transmission expansion planning formulation presented in this paper is characterized by a complete decoupling between real and reactive power equations. The flowchart, shown in Fig. 1, summarizes the planning algorithm.

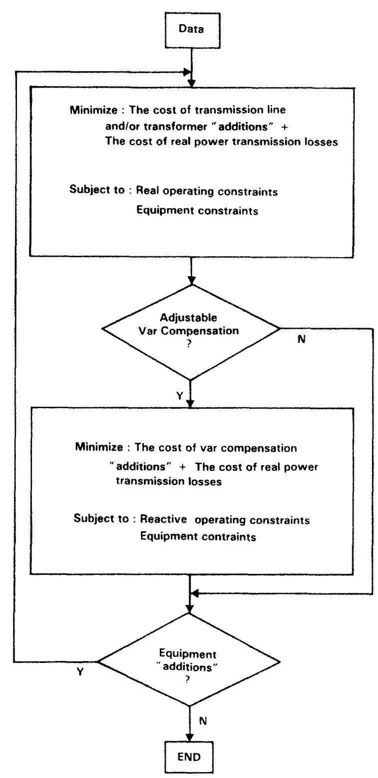

Fig. 1 Simplified Flowchart of the Planning Algorithm.

Discrete sensitivity formulations have been proposed in the past [1-7] for solving the single-stage transmission expansion planning problem. However these formulations suffer from two major drawbacks:

1. The need for a connected power system.
2. The inital treatment of the design parameters as continuous variables.

Indeed, in these formulations the line parameters are first linearized. Then the increments are treated as discrete variables to reflect the discrete nature of equipment additions. Figure 2 compares the discrete linear sensitivity formulation to the exact equations

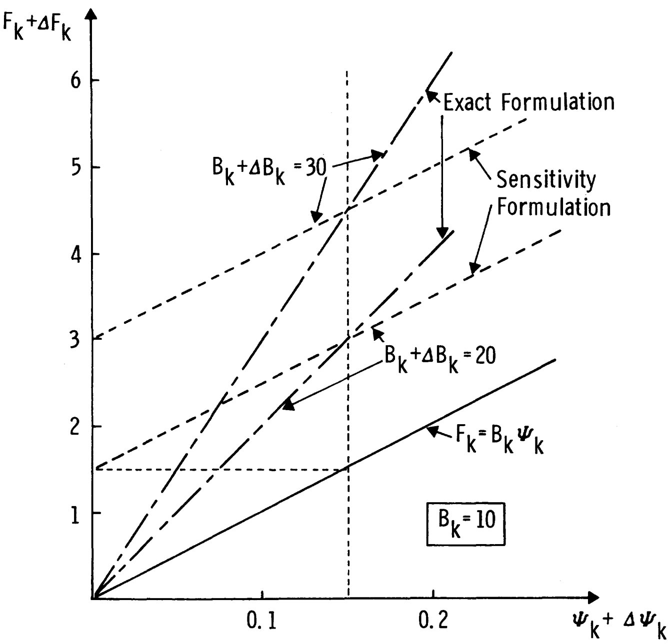

Fig. 2 Plots Comparing the Discrete Linear Sensitivity Formulation and the Implicit Enumeration Formulation.

when the right-of-way is considered for reinforcements and its line equation is given by

$$F_k = B_k \psi_k$$

where F_k is the real-power flow, B_k is the line susceptance, and ψ_k is the angle difference across the line. For this particular model equation the new formulation that is described in this paper, called the implicit enumeration formulation, coincides with the exact equations.

The main point to observe in Fig. 2 is that the two formulations give the same real-power flow only at $\Delta\psi_k = 0$. However, for values of $\Delta\psi_k \neq 0$, the real-power flows are substantially different for a given ΔB_k. This difference increases even more as ΔB_k increases. Therefore, unless $(\Delta B_k/B_k)$ and $(\Delta\psi_k/\psi_k)$ remain very small, the discrete linear sensitivity formulation is quite inaccurate.

Quadratic sensitivity formulation as proposed in reference [7] would offer a comparable accuracy with the implicit enumeration formulation. However, as stated in reference [7], it still needs a connected system. Moreover, such quadratic sensitivity formulation uses nonlinear integer programming techniques, whereas the implicit enumeration formulation uses the much simpler linear integer programming techniques.

In the following sections, the "real" planning equations, together with the new implicit enumeration formulation, and the "reactive" planning equations are described. Some examples, one of which provides a solution not obtainable with linear discrete sensitivity formulation, are presented at the end of this paper.

The ac-network model equations used throughout this paper are described in Appendix A.

THE "REAL" PLANNING EQUATIONS

Ac Right-of-Way Planning Equations - The Implicit Enumeration Formulation

Holding voltage and transformer tap magnitudes constant, a first-order approximation, with respect to the base case, of the ac-model equations (A-1) and (A-2) for each configuration that can be given to the right-of-way k is given by the following sets of equations

$$F_k + \Delta F_k = 0 \tag{1a}$$

$$P_{\ell,k} + \Delta P_{\ell,k} = 0 \tag{1b}$$

for the empty configuration, or

$$F_k + \Delta F_k = G_{k1}V_kU_k + B_{k1}V_k^2 (\psi_k + \Delta\psi_k) \tag{1c}$$

$$P_{\ell,k} + \Delta P_{\ell,k} = G_{k1} (U_k^2 + V_k^2 \psi_k^2) + 2G_{k1}V_k^2\psi_k\Delta\psi_k \tag{1d}$$

for the nonempty configuration #1, or etc.

Introducing some auxiliary variables and using integer variables, the implicit enumeration formulation combines these equations as follows

$$\Delta F_k = \sum_{r=1}^{s} B_{kr}V_k^2\Delta\psi_{kr} + \sum_{r=1}^{s} (G_{kr}V_kU_k + B_{kr}V_k^2\psi_k) W_{kr} - F_k \tag{2a}$$

$$\Delta P_{\ell,k} = \sum_{r=1}^{s} 2G_{kr}V_k^2\psi_k\Delta\psi_{kr} + \sum_{r=1}^{s} G_{kr} (U_k^2 + V_k^2\psi_k^2) W_{kr} - P_{\ell,k} \tag{2b}$$

$$\Delta\psi_k = \sum_{r=0}^{s} \Delta\psi_{kr} \tag{2c}$$

$$-\psi_{kr}^M W_{kr} \leq \psi_k W_{kr} + \Delta\psi_{kr} \leq \psi_{kr}^M W_{kr} \qquad r = 0,\dots,s \tag{2d}$$

$$\sum_{r=0}^{s} W_{kr} = 1 \tag{2e}$$

$$0 \leq W_{kr} \leq 1 \qquad r = 0,\dots,s \tag{2f}$$

where G_{kr}, B_{kr}: are, respectively, the equivalent conductance and the equivalent susceptance corresponding to the rth configuration in right-of-way k.

W_{kr}: is an integer variable that takes the value 1 when the rth configuration available in right-of-way k is chosen, and takes the value 0 otherwise.

$\Delta\psi_{kr}$: is an auxiliary variable that is equal to $\Delta\psi_k$ when the rth configuration in right-of-way k is chosen, and that is equal to zero otherwise.

ψ_{kr}^M: is the emergency stability rating of the rth configuration in right-of-way k. In the case of an empty configuration (i.e r = 0), this upper-bound is chosen large enough to deactivate equation (2d).

Note that r can take a value equal to 0 only if an empty configuration is available in right-of-way k, such that s is the number of non-empty configurations available in right-of-way k.

When the configuration in right-of-way k is fixed, this formulation provides the same equations as the discrete linear sensitivity formulation. However, if the right-of-way k can be equipped or reinforced, this formulation provides much more accurate equations than those produced by the discrete linear sensitivity method. Whatever configuration is chosen in the right-of-ways, the accuracy of this new formulation is only limited by the accuracy of the ac-network model equations [8-10]. Contrary to the discrete linear sensitivity formulation, this new formulation does not linearize the design parameters. Notice that only the real transmission loss equation is linearized about the base-case solution. Therefore, even if sensitivity coefficients were not available, the real-power flow equation is still applicable. The real-power flow equations produced by the two formulations are compared with the exact real-power flow equations in Fig. 3, assuming that the variations in voltage and transformer tap magnitudes can be neglected. The relative errors in the two formulations are plotted for different values of $\Delta\psi_k$ when a second line is build in the right-of-way. This example shows clearly that the discrete linear sensitivity formulation is simply unacceptable unless the variations are expected to be very small.

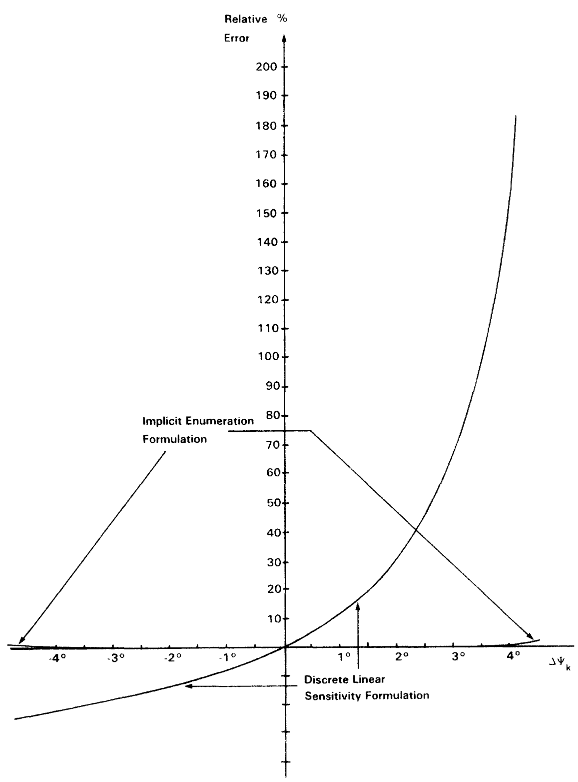

Fig. 3 Plots Comparing the Accuracy of the Discrete Linear Sensitivity Formulation and the Implicit Enumeration Formulation.

The real power flow rating in right-of-way k is dependent upon the chosen configuration, thus the real power flow must satisfy (See Appendix A)

$$-\sum_{r=0}^{s} P^M_{kr} W_{kr} \leqq (F_k - P_{\ell,k}/2) + \Delta F_k - \Delta P_{\ell,k}/2 \qquad (3a)$$

$$(F_k + P_{\ell,k}/2) + \Delta F_k + \Delta P_{\ell,k}/2 \leqq \sum_{r=0}^{s} P^M_{kr} W_{kr} \qquad (3b)$$

where P^M_{kr} is the real power flow rating in right-of-way k corresponding to the rth configuration (Note: $P^M_{ko} = 0$).

The emergency stability ratings in right-of-way k are included in equation (2d).

If one or more configuration in right-of-way k contain a phase shifter, the following constraints must also be satisfied

$$-\sum_{r=0}^{s} \alpha^m_{kr} W_{kr} \leqq \alpha_k + \Delta\alpha_k \leqq \sum_{r=0}^{s} \alpha^M_{kr} W_{kr} \qquad (4)$$

where α^m_{kr} is minus the minimum phase shift corresponding to the rth configuration ($\alpha^m_{ko} = 0$) in the right-of-way k, and α^M_{kr} is the maximum phase shift corresponding to the rth configuration ($\alpha^M_{ko} = 0$) in the same right-of-way.

Upon substituting equations (2a) and (2b), the inequality constraints given by equations (3) and (2d) can be combined into the following reduced set of inequality constraints

$$-\psi'^m_{kr} W_{kr} \leqq \Delta\psi_{kr} \leqq (\psi'^M_{kr} - \psi'^m_{kr}) W_{kr}$$
$$r = 0,\ldots,s \qquad (5)$$

where
$$\psi'^m_{ko} = \psi^M_{ko} + \psi_k \qquad (6a)$$

$$\psi'^m_{kr} = \min\{(\psi^M_{kr} + \psi_k);\ [P^M_{kr} + (G_{kr}V_kU_k + B_{kr}V_k^2\psi_k) - G_{kr}(U_k^2 + V_k^2\psi_k^2)/2]/(B_{kr}V_k^2 - G_{kr}V_k^2\psi_k)\}$$
$$r=1,\ldots,s \qquad (6b)$$

$$\psi'^M_{ko} = 2\psi^M_{ko} \qquad (6c)$$

$$\psi'^M_{kr} = \min\{(\psi^M_{kr} - \psi_k);\ [P^M_{kr} - (G_{kr}V_kU_k + B_{kr}V_k^2\psi_k) - G_{kr}(U_k^2 + V_k^2\psi_k^2)/2]/(B_{kr}V_k^2 + G_{kr}V_k^2\psi_k)\} + \psi'^m_{kr} \qquad r = 1,\ldots,s \qquad (6d)$$

In order to have all non-negative variables, we define the following auxiliary variables

$$\Delta\psi'_{kr} = \Delta\psi_{kr} + \psi'^m_{kr} \qquad r=0,\ldots,s \qquad (7a)$$

$$\Delta\alpha'_k = \Delta\alpha_k + \alpha_k + \sum_{r=0}^{s} \alpha^m_{kr} W_{kr} \qquad (7b)$$

Using equations (7a), (7b), (2c) and (2e) to eliminate, respectively, $\Delta\psi_{kr}$ (r=0,...,s), $\Delta\alpha_k$, $\Delta\psi'_{ks}$ and W_{ks}, equations (2a), (2b), (5), (4) and (2f) become, respectively,

$$\Delta F_k = B_{ks}V_k^2\Delta\delta_i - B_{ks}V_k^2\,\Delta\delta_j - B_{ks}V_k^2\Delta\alpha'_k + \sum_{r=0}^{(s-1)} (B_{kr} - B_{ks})\,V_k^2\Delta\psi'_{kr} + \sum_{r=0}^{(s-1)} [(G_{kr} - G_{ks})\,V_kU_k + (B_{kr} - B_{ks})\,V_k^2(\psi_k - \psi'^{m}_{kr}) + B_{ks}\,V_k^2\,(\alpha^m_{kr} - \alpha^m_{ks})]\,W_{kr} + [G_{ks}V_kU_k + B_{ks}\,V_k^2\psi_k + B_{ks}V_{ks}^2\,(\alpha^m_{ks} + \alpha_k) - F_k] \tag{8a}$$

$$\Delta P_{\ell,k} = 2G_{ks}V_k\psi_k\Delta\delta_i - 2G_{ks}V_k^2\psi_k\Delta\delta_j - 2G_{ks}V_k^2\psi_k\Delta\alpha'_k + \sum_{r=0}^{(s-1)} 2(G_{kr} - G_{ks})\,V_k^2\psi_k\Delta\psi'_{kr} + \sum_{r=0}^{(s-1)} [(G_{kr}-G_{ks})\,(U_k^2 + V_k^2\psi_k^2 - 2\psi_k\,\psi'^{m}_{kr}) + 2G_{ks}\,V_k^2\,\psi_k\,(\alpha^m_{kr} - \alpha^m_{ks})]\,W_{kr} + [G_{ks}\,(U_k^2+V_k^2\psi_k^2) + 2G_{ks}\,V_k^2\psi_k\,(\alpha^m_{ks}+\alpha_k) - P_{\ell,k}] \tag{8b}$$

Subject to:

$$0 \leqq \Delta\delta_i - \Delta\delta_j - \Delta\alpha'_k - \sum_{r=0}^{(s-1)} \Delta\psi'_{kr} + \sum_{r=0}^{(s-1)} [(\psi'^{m}_{kr}-\psi'^{m}_{ks}) + (\alpha^m_{kr}-\alpha^m_{ks})]\,W_{kr} + (\psi'^{m}_{ks}+\alpha^m_{ks}+\alpha_k) \leqq -\sum_{r=0}^{(s-1)} \psi'^{M}_{ks}\,W_{kr} + \psi'^{M}_{ks} \tag{9a}$$

$$0 \leqq \Delta\psi'_{kr} \leqq \psi'^{M}_{kr}\,W_{kr} \qquad r = 0,\ldots,(s-1) \tag{9b}$$

$$0 \leqq \Delta\alpha'_k \leqq \sum_{r=0}^{(s-1)} [(\alpha^M_{kr} - \alpha^M_{ks}) + (\alpha^m_{kr} - \alpha^m_{ks})]W_{kr} + (\alpha^M_{ks} + \alpha^m_{ks}) \tag{9c}$$

$$0 \leqq W_{kr} \leqq 1 \qquad r = 0,\ldots,(s-1) \tag{9d}$$

$$\sum_{r=0}^{(s-1)} W_{kr} \leqq 1 \tag{9e}$$

$$W_{kr} = 0 \text{ or } 1 \qquad r=0,\ldots,(s-1) \tag{9f}$$

A comparison between these equations and the equations derived from the discrete linear sensitivity formulation [9] shows an increase in the number of constraints and variables. This increase lies in the constraint equations (9b). In order to minimize the number of constraints and variables and to maximuze the accuracy, the discrete linear sensitivity formulation and the implicit enumeration formulation are combined in the planning formulation presented in this paper. The problem of allocating a formulation to a right-of-way is discussed later after some examples are presented. Fixed right-of-ways are described in Reference [9] and involve only one equality constraint.

The Ac-Node Planning Equations

Let us define an ac-subarea to be a connected digraph consisting of a non-empty set of ac-buses and a non-empty set of ac right-of-ways. Let us also define the planning ac-transmission network to be a connected digraph consisting of a non-empty set of ac-buses (existing and new) and a non-empty set of ac right-of-ways (existing and new). Finally, let us define the interconnecting right-of-ways to be the right-of ways interconnecting the base-case ac-subareas in the planning ac-transmission network. According to the previous discussions, the implicit formulation is always used in these interconnecting right-of-ways.

In the planning ac-transmission network we can write the following relation, in matrix form,

$$[\Delta\psi] + [\Delta\alpha] = [A]^t\,[\Delta\delta] \tag{10}$$

where [A]: is the reduced ac-incidence matrix.

[$\Delta\psi$]: is the vector of changes in the angle differences across the right-of-ways.

[$\Delta\alpha$]: is the vector of changes in the phase-shifts in the right-of-ways.

[$\Delta\delta$]: is the vector of changes in the ac-bus voltage angles.

For simplicity, let us rewrite the right-of-way equations of both formulations in the common compact form

$$\Delta F_k = C_kV_k^2\Delta\delta_i - C_kV_k^2\Delta\delta_j + \Delta x_k \tag{11a}$$

$$\Delta P_{\ell,k} = 2D_kV_k^2\psi_k\Delta\delta_i - 2D_kV_k^2\psi_k\Delta\delta_j + 2\,\Delta y_k \tag{11b}$$

In matrix form, these equations become

$$[\Delta F] = d(CV^2)\,\{[\Delta\psi] + [\Delta\alpha]\} + [\Delta x] \tag{12a}$$

$$[\Delta P_\ell] = 2d(DV^2\psi)\,\{[\Delta\psi] + [\Delta\alpha]\} + 2\,[\Delta y] \tag{12b}$$

where d(·) stands for diagonal matrix.

Excluding the slack-bus real-power equation, the node equations in the planning ac-transmission network are, in matrix form, given by (see Appendix A).

$$[\Delta P] = [A]\,[\Delta F] + \tfrac{1}{2}\,|[A]|\,[\Delta P_\ell] \tag{13}$$

Upon substituting equations (10) and (12), equation (13) becomes

$$[\Delta P] = [J]\,[\Delta\delta] + [A]\,[\Delta x] + |[A]|\,[\Delta y] \tag{14}$$

$$\text{where } [J] = \{[A]\,d(CV^2) + |[A]|\,d(DV^2\psi)\}\,[A]^t \tag{15}$$

Solving equation (14) for [$\Delta\delta$] and substituting into equation (10), we obtain for every right-of-way k in the planning ac-transmission network

$$\Delta\delta_i - \Delta\delta_j = -\sum_{e=1}^{N-1} (K'_{je} - K'_{ie})\;\Delta P_e + \sum_{\ell} [(K'_{jm} - K'_{im}) - (K'_{jn} - K'_{in})]\,\Delta x_\ell + \sum_{\ell} [(K'_{jm} - K'_{im}) + (K'_{jn} - K'_{in})]\,\Delta y_\ell \tag{16}$$

where N: is the number of ac-buses in the planning ac-transmission network.

ℓ: is any ac right-of-way connected between ac-buses m and n.

$$K'_{ij} = \begin{cases} \{[J]^{-1}\}_{ij} & \text{if } i \text{ and } j < N \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

Finally, equation (16) is substituted into the right-of-way equation corresponding to the real transmission losses and into the right-of-way constraint equations.

The Investment Costs

Fixed right-of-ways do not incur any investment cost. Right-of-ways that can be equipped or reinforced incur some investment costs which are independent of the formulation used. These investment costs are given by

$$I\$ = \sum_k \{ \sum_{r=0}^{(s-1)} (I_{kr} - I_{ks}) W_{kr} + I_{ks}\} \quad (18)$$

where I_{kr}: is the investment cost [10] associated with the rth configuration in right-of-way k.

The Operating Cost

The operating cost is given by the cost of real ac-transmission losses. Thus, for the ac-transmission network, we have

$$O\$ = \sum_k \{L (P_{\ell,k} + \Delta P_{\ell,k})\} \quad (19)$$

where L: is the unit cost of real ac-transmission losses.

THE "REACTIVE" PLANNING EQUATIONS

The "reactive" equations are derived to help containing the investment costs. By maintaining a reasonable voltage level in the planned ac-transmission network, high reactive power flows that reduce the real capacity of right-of-ways can be avoided. This will allow for a better use of the real capacity of additional lines that may be needed in some right-of-ways. Since var compensations are less expensive than line additions it will help maintaining the investment costs to their minimum. In these equations transformer taps and reactive generations from ac-generators are considered operating tools, and therefore, the only planning tools involved in this formulation are local var compensations from capacitor or inductor banks. The locations where these banks can be placed are assumed given. The optimal var allocation is derived using a discrete linear sensitivity formulation to account for the discrete nature of var compensations. The resulting allocation is optimal in the sense that investment costs in terms of equipment additions and operating costs in terms of real-power transmission losses are minimized. The right-of-way configurations corresponding to the optimal solution of the "real" planning equations are not affected in these "reactive" planning equations.

Ac Right-of-Way Planning Equations

Holding transformer taps magnitudes constant, a first order approximation of equations (A-1b), (A-2b) and (A-2a) with respect to the base-case is given by

$$\Delta T_k = [(B_k/t_k)(V_i/t_k) - (G_k/t_k)V_k\psi_k]\Delta V_i - [B_k V_j + G_k V_k \psi_k]\,\Delta V_j \quad (20a)$$

$$\Delta Q_{\ell,k} = (B_k/t_k)(2U_k + V_k\psi_k^2)\,\Delta V_i - B_k(2U_k - V_k\psi_k^2)\,\Delta V_j \quad (20b)$$

$$\Delta P_{\ell,k} = (G_k/t_k)(2U_k + V_k\psi_k^2)\,\Delta V_i - G_k(2U_k - V_k\psi_k^2)\,\Delta V_j \quad (20c)$$

where Δ indicates the change with respect to the base-case values.

The Ac-Node Planning Equations

A first order approximation of the reactive node equations (A-3b) are, in matrix form, given by

$$[\Delta Q] = [A]\,[\Delta T] + \tfrac{1}{2}\,|[A]|\,[\Delta Q_\ell] - d\,(2B_c V)\,[\Delta V] - d(V^2)\,[\Delta B_c] - d\,(B^c V)\,[\Delta V] \quad (21)$$

where Δ indicates the change with respect to the base-case values and

$$B_e^c = \sum_k |A_{ek}|\, B_k^c \quad (22)$$

Upon substituting equations (20a) and (20b), equation (21) becomes

$$[\Delta Q] = [H]\,[\Delta V] - d\,(V^2)\,[\Delta B_c] \quad (23)$$

To guarantee a proper voltage level in the planned ac-transmission network, the following constraints must be satisfied at every bus

$$V_e^m \leqq V_e + \Delta V_e \leqq V_e^M \quad (24)$$

At some buses, the net injected reactive power is continuously adjustable within some bounds. Thus, the following constraints must also be satisfied at these buses

$$Q_e^m \leqq Q_e + \Delta Q_e \leqq Q_e^M \quad (25)$$

Additional var compensations in the form of shunt capacitors or shunt inductors are available at specified locations in the ac-transmission network. Since these var compensations are usually available in discrete units, we have at bus e

$$\Delta B_{c,e} = \sum_{r=0}^{s} (B_{c,er} - B_{c,e})\, W_{c,er} \quad (26a)$$

$$\sum_{r=0}^{s} W_{c,er} = 1 \quad (26b)$$

$$0 \leqq W_{c,er} \leqq 1 \qquad r = 0,\dots,s \quad (26c)$$

$$W_{c,er} = 0 \text{ or } 1 \qquad r = 0,\dots,s \quad (26d)$$

where $B_{c,er}$: is the shunt susceptance corresponding to the rth combination of shunt available at bus e ($B_{c,eo} = 0$).

$W_{c,er}$: is an integer variable with a value equal to 1 when the rth combination of shunt available at bus e is chosen, and has a value equal to 0 otherwise.

In order to have all non-negative variables, we define the following auxiliary variables

$$\Delta V'_e = \Delta V_e + (V_e - V_e^m) \tag{27a}$$

$$\Delta Q'_e = \Delta Q_e + (Q_e - Q_e^m) \tag{27b}$$

Using equations (26b), (27a) and (27b) to eliminate, respectively, $W_{c,es}$, ΔV_e and ΔQ_e, equations (23), (26c), (24), and (25) become, respectively, for bus i

$$\Delta Q'_i - \sum_{j=1}^{N} H_{ij} \Delta V'_j + \sum_{r=0}^{s} (B_{c,ir} - B_{c,i}) V_i^2 W_{c,ir} = Q_i - Q_i^m - \sum_{j=1}^{N} H_{ij} (V_j - V_j^m) \tag{28a}$$

$$\sum_{r=0}^{(s-1)} W_{c,ir} \leqq 1 \tag{28b}$$

$$0 \leqq W_{c,ir} \leqq 1 \qquad r=0,\ldots,(s-1) \tag{28c}$$

$$0 \leqq \Delta V'_i \leqq (V_i^M - V_i^m) \tag{28d}$$

$$0 \leqq \Delta Q'_i \leqq (Q_i^M - Q_i^m) \tag{28e}$$

$$W_{c,ir} = 0 \text{ or } 1 \qquad r=0,\ldots,(s-1) \tag{28f}$$

Notice that equation (26a) corresponds to a discrete linear sensitivity approach. An implicit enumeration can be applied [8]. However, the relative error produced by equation (26a) is in most cases much smaller than 10%. This, simply because $\Delta V_e/V_e$ is in most cases much smaller than 10%. In addition, the iterative nature of the combined planning formulations will guarantee the necessary accuracy. Therefore the improvements in the accuracy using an implicit enumeration formulation can not be important enough to justify the increase in the number of constraints.

The Investment Costs

Additional var compensations incur some investment costs given by

$$I\$ = \sum_{i=1}^{N} \left\{ \sum_{r=0}^{(s-1)} (I_{c,ir} - I_{c,is}) W_{c,ir} + I_{c,is} \right\} \tag{29}$$

where $I_{c,ir}$: is the investment cost associated with the rth combination of shunts connected to bus i.

The Operating Cost

The operating cost is given by the cost of real ac-transmission losses, i.e.,

$$O\$ = \sum_{k} \{L\,(P_{\ell,k} + \Delta P_{\ell,k})\} \tag{30}$$

where L: is the unit cost of real ac-transmission losses.

IMPLEMENTATION OF THE AC-TRANSMISSION EXPANSION PLANNING ALGORITHM

A simplified flowchart of the optimal transmission network expansion algorithm is shown in Fig. 4. A description of these programs is given in Reference [10].

The Six Bus Garver Example [2]

Figure 5 illustrates the ac-network with its future generation and load locations. Note in Fig. 5 that a new generating plant (bus number 6) is initially not connected to the original five-bus system. The load-flow data corresponding to the initial network and the planning data are given in References [2] and [9]. This example is presented to illustrate that the method proposed is capable of handling disconnected networks without the necessity to create an initial connected network. In this example there is no line-charging and no var compensation.

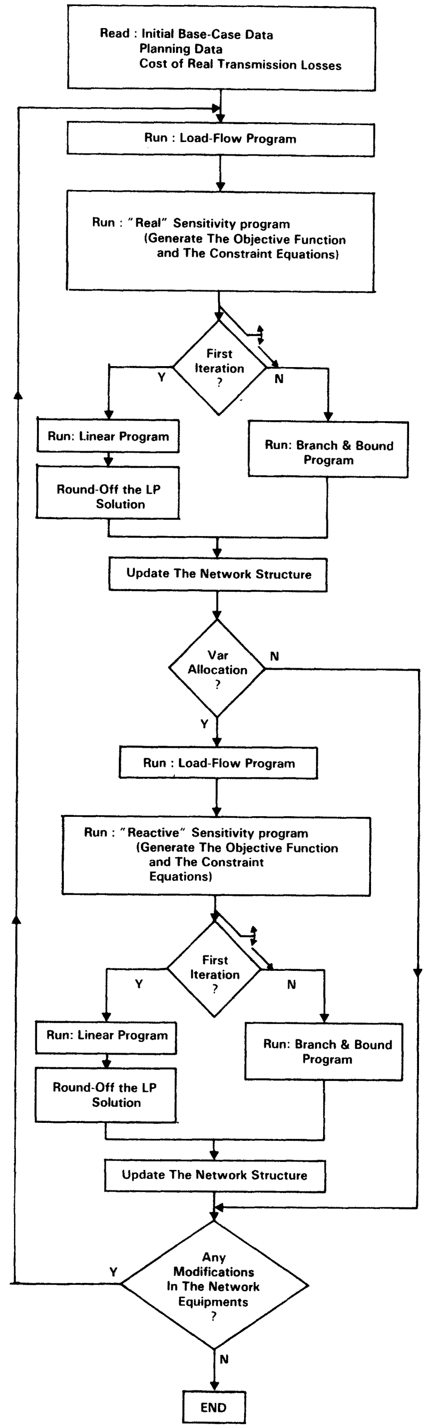

Fig. 4. Simplified Flowchart of the Optimal Transmission Network Planning Algorithm

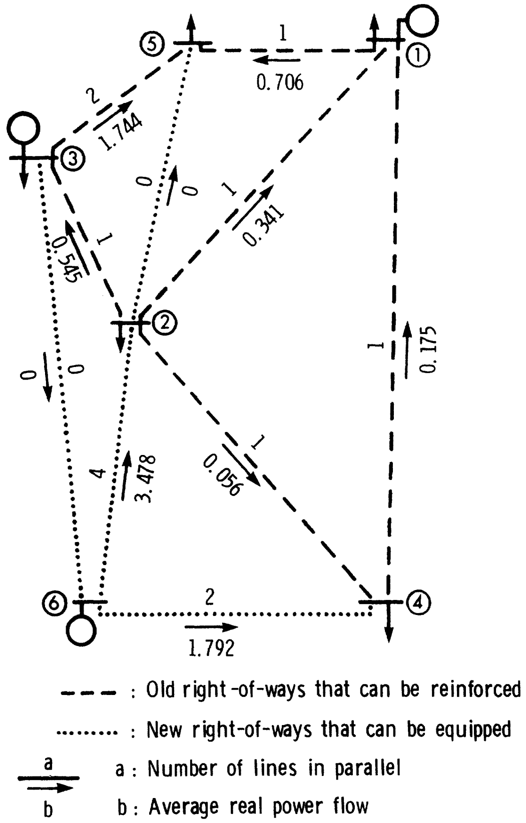

Fig. 5. Optimal Transmission Plan for the Six-Bus Garver System

The optimal network expansion plan minimizes only the investment cost, thus the cost of real transmission losses is set equal to zero. The implicit enumeration formulation is chosen for the right-of-ways that interconnect the new generating plant to the initial five-bus system, and the discrete linear sensitivity formulation is chosen for the other right-of-ways. The optimal solution (second iteration) is shown in Fig. 5. The problem required 1.5 seconds of CPU-time (including the third check iteration).

The 39-Bus New England Example

Figure 6 illustrates a geographic map of the system with the right-of-way length approximately proportional to the line reactances. Bus and line data, for the initial network, are given in Reference [9]. These data are substantially modified from the original data in references [11-13]. Six right-of-ways have been added to the original system. The planning bus data involving a shift in generation levels are also given in Reference [9]. This shift in supply causes overloads on the five transmission lines (1-2), (2-3), (23-24), (25-26), and (6-23). The planning line data for the right-of-ways that can be equipped or reinforced are described in Reference [9]. This example is presented to illustrate a case where the inaccuracies of the discrete linear sensitivity formulation result in no solution being obtained, and where the implicit enumeration formulation is needed to provide a solution.

Assuming no var-compensation, we consider an ac-expansion plan that will minimize only the investment costs. The implicit enumeration formulation is chosen for all the right-of-ways that can be equipped or reinforced. The second alternative numbers in Fig. 6 represent the optimal solution obtained after three iterations. The problem required 3.7 minutes of CPU-time (including the fourth check iteration).

What is most interesting in this example is the lack of a feasible solution using the discrete linear sensitivity formulation. As this example has not been specially designed to reach this observation, this situation may not be unlikely.

ALLOCATION OF THE TWO FORMULATIONS

It is clear now that the implicit enumeration formulation will always be used with the interconnecting right-of-ways.

A first criterion to allocate the two formulations between the right-of-ways consists in using the discrete linear sensitivity formulation with the right-of-ways that have a few admissible alternatives and the implicit enumeration formulation with the right-of-ways that have a relatively large number of admissible alternatives.

Further research is in course to derive better criterions that will allow a more optimal allocation between the two formulations. One such work involves a close look at the "dc" sensitivity matrix ($\partial F/\partial B$) [1,4]. However, more tests are still necessary to reach any general conclusions.

CONCLUSIONS

A decoupled single-stage ac-transmission expansion planning formulation has been developed to produce an optimal plan for adding ac transmission lines and var compensations. The optimality of the plan is guaranteed by minimizing the total cost including the investment and the operating costs.

The main contribution is a new formulation called the implicit enumeration formulation that allows an exact treatment of the discrete nature of equipment additions, and still uses mixed integer linear programming. It also allows the interconnection of disconnected systems without resorting to an initial connected system.

In addition, the complete set of admissible network reinforcements is included; linear and discrete cost-capacity curves can be used; and a new efficient decoupled linear load-flow model is incorporated.

Two examples have been incorporated to illustrate the potentials of this new optimum planning formulation. The advantages of this new formulation over the discrete linear sensitivity formulation are clearly ascertained from these two examples.

Future research work need to investigate the sensitivity of the optimum configuration to var compensations and to variations in the unit operating cost. Additional work need also to develop some criterions to optimally combine the discrete linear sensitivity formulation and the implicit enumeration formulation.

ACKNOWLEDGEMENTS

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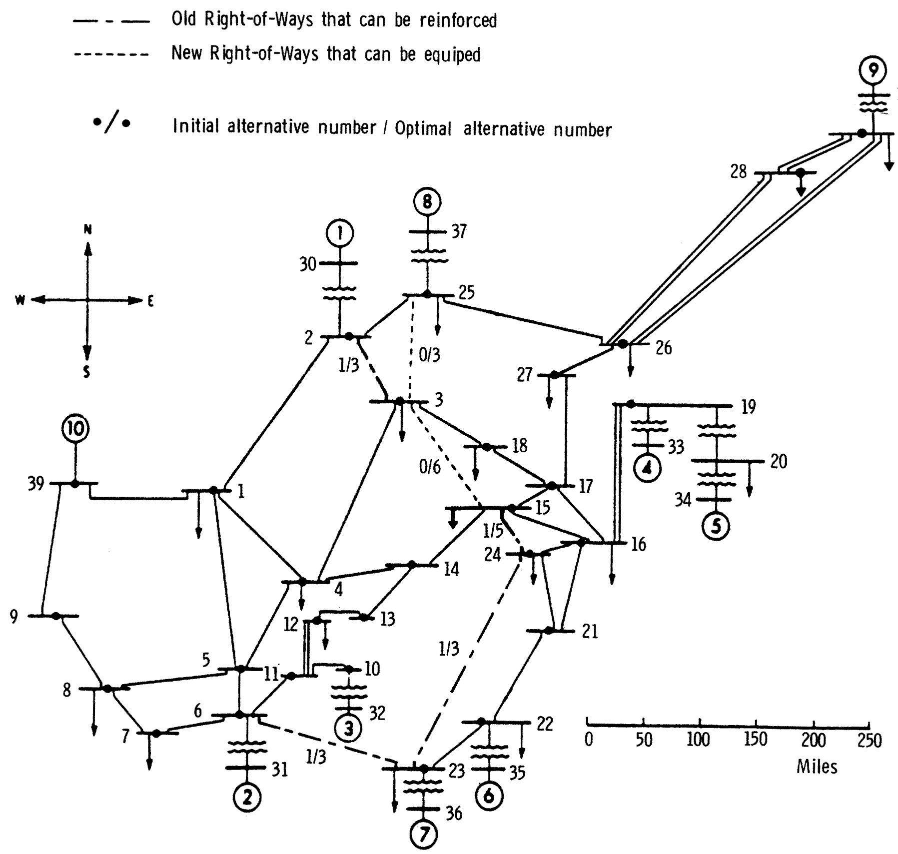

Fig. 6. New England 39-Bus Test System Map

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APPENDIX A

Figure 7 illustrates the equivalent model of an ac-transmission branch representing either a transformer or a transmission line in right-of-way k. The branch is oriented from bus-i to bus-j. The average real and reactive power flows are, respectively, approximated by [8-10]

$$F_k = G_k V_k U_k + B_k V_k^2 \psi_k \qquad \text{(A-1a)}$$

$$T_k = B_k V_k U_k - G_k V_k^2 \psi_k \qquad \text{(A-1b)}$$

where F_k: is the average real power flowing from bus-i to bus-j, thus $F_k = (P_{ij} - P_{ji})/2$.

T_k: is the average reactive power flowing from bus-i to bus-j, thus $T_k = (Q_{ij} - Q_{ji})/2$.

G_k, B_k: are, respectively, the equivalent line conductance and the equivalent line susceptance such that the equivalent line admittance is given by $\tilde{Y}_k = G_k - jB_k$.

V_k: is the average line voltage for branch-k defined by $V_k = (V_i/t_k + V_j)/2$.

U_k: is the voltage difference for branch-k defined by $U_k = (V_i/t_k - V_j)$.

ψ_k: is the angle difference for branch-k defined by $\psi_k = \delta_i - \alpha_k - \delta_j$.

The real and reactive power losses in the same right-of-way k are, respectively, approximated by the following equations [8-10]

$$P_{\ell,k} = G_k \, (U_k^2 + V_k^2 \, \psi_k^2) \qquad \text{(A-2a)}$$

$$Q_{\ell,k} = B_k \, (U_k^2 + V_k^2 \, \psi_k^2) \qquad \text{(A-2b)}$$

where $P_{\ell,k}$: is the real power loss in right-of-way k, thus $P_{\ell,k} = P_{ij} + P_{ji}$.

$Q_{\ell,k}$: is the reactive power loss in right-of-way k, thus $Q_{\ell,k} = Q_{ij} + Q_{ji}$.

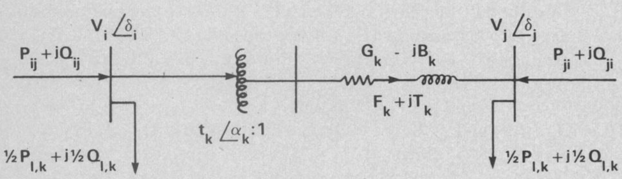


Fig. 7. Equivalent Model for an Ac-Transmission Branch in Right-of-Way k (Line charging are included in the node equations).

Then the net injections of real and reactive powers into bus-e are, respectively, given by

$$P_e = \sum_k A_{ek} F_k + \tfrac{1}{2} \sum_k |A_{ek}| P_{\ell,k} \qquad \text{(A-3a)}$$

$$Q_e = - B_{c,e} V_e^2 + \sum_k A_{ek} T_k + \tfrac{1}{2} \sum_k |A_{ek}| Q_{\ell,k}$$

$$- V_e^2 \sum_k |A_{ek}| \, (B_k^c/2) \qquad \text{(A-3b)}$$

where P_e: is the net injection of real power into bus-e, thus $P_e = P_{ge} - P_{de}$.

Q_e: is the net injection of reactive power into bus-e, thus $Q_e = Q_{ge} - Q_{de}$.

$B_{c,e}$: is the shunt susceptance connected to bus-e such that the shunt admittance is given by $\tilde{Y}_{c,e} = j \, B_{c,e}$.

V_e: is the voltage magnitude at bus-e.

B_k^c: is the line charging susceptance in the right-of-way k.

A_{ek}: is the element in the eth row and the kth column of the ac-incidence matrix.

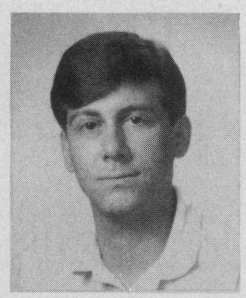
$|A_{ek}|$: is the absolute value of A_{ek}.

A necessary and sufficient condition to satisfy the rating of right-of-way k at both extremities is expressed by the two following inequality constraints

$$F_k - P_{\ell,k}/2 \geqq - P_k^M \qquad \text{(A-4a)}$$

$$F_k + P_{\ell,k}/2 \leqq P_k^M \qquad \text{(A-4b)}$$

where P_k^M is the maximum real-power flow admissible in right-of-way k.



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